ALL(Except CSE & IT)

Jharkhand University of Technology, Ranchi

B.Tech. 2nd Semester Examination, 2019

(Held in May, 19)

Subject : Mathematics-II **Subject Code : BSC-104**

Time Allowed : 3 Hours **Full Marks : 70**

Candidates are required to give their answers in their own words as far as practicable.

The figures in the right hand margin indicate full marks.

Answer any five *questions.*

1. There are SEVEN multiple choice questions. Each questions has 4 choices (a), (b), (c) and (d), out of which only one is correct. Choose the correct answer: 2×7=14

(i) Change the order of integration in double integral $I = \int_0^8 \int_{\frac{x}{4}}^2 f(x,y)dydx$ leads to $I = \int_r^s \int_p^q f(x,y)dxdy$. What is q and s?

(a) $4y$ and 4 (b) $4y$ and 2

(c) $2y$ and 2 (d) $2y$ and 4

(ii) $\iint_s (\nabla \times F).Nds$, where F is a vector, is equal to

(a) $\oint_C (\nabla \times F).dR$ (b) $\oint_C \nabla.F.dR$

(c) $\oint_C F.dR$ (d) $\oint_C (\nabla \times \nabla \times F).dR$

(iii) $\frac{dy}{dx} = \tan\left(y - x\frac{dy}{dx}\right)$ then,

(a) $y = cx + \tan(cx)$ (b) $y = cx + \tan^{-1}(c)$

(c) $y = c\tan x + \tan^{-1} x$ (d) $y = cx + \tan^{-1} x$

(iv) Find the value of k, if the following differential equation is exact.

$(xy^2 + kx^2y)dx + (x+y)x^2dy = 0$

(a) $k = 1$ (b) $k = 2$

(c) $k = 3$ (d) $k = 4$

(v) Find the particular integral of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 3e^{2x}$

(a) $e^{3x}/3$ (b) $e^{2x}/3$

(c) $e^{3x}/5$ (d) $e^{2x}/5$

(vi) Find fixed points of the bilinear mapping $w = \frac{z-1}{z+1}$

(a) $1, -1$ (b) $i, -i$

(c) $1, i$ (d) $1, -i$

(vii) The value of $\oint_C (z-z_0)^m dz$ where C is the unit circle $|z-z_0| = r, z_0 \in \mathbb{C}, r \in \mathbb{R}$ are constants and $m \neq -1$ an integer.

(a) $2\pi i$ (b) $-2\pi i$

(c) 2π (d) 0

Or,

(a) Change the order of integration in the double integral $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2a}} f(x,y)dxdy$

(b) Calculate by double integration the volume generated by the revolution of the cardioid $r = a(1-\cos\theta)$ about its axis.

(c) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

4+5+5=14

2. (a) If $\vec{F} = 3xy\,\hat{\imath} - y^2\hat{\jmath}$, evaluate $\int_C \vec{F}.d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from (0,0) to (1,2).

(b) Find $\iint_S \vec{F}.\hat{n}\, dS, \vec{F} = (2x+3z)\hat{\imath} - (xz+y)\hat{\jmath} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having centre at (3, –1, 2) and radius 3.

14 (c) Evaluate $\oint_C \vec{F}.d\vec{r}$ by Stokes's theorem, where $\vec{F} = y^2\hat{\imath} + x^2\hat{\jmath} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at (0,0,0), (1,0,0) and (1,1,0).

4+5+5=14

3. (a) Solve $x\left[\frac{dx}{dy} + y\right] = 1 - y$

(b) Solve $y = 2px + y^2p^2$, where p denotes $\frac{dy}{dx}$.

10 (c) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

4+5+5=14

4. (a) Solve $(D^2 - 4D + 3)y = e^x \cos 2x$, where $D = \frac{d}{dx}$.

(b) Apply the method f variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$

10 x 4 (c) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$

4+5+5=14

$x = e^t$

10

5. (a) Find the Frobenius series solution about $x = 0$, of the differential equation

$$(1-x^2)y'' - 2xy' + 6y = 0$$

(b) If $J_n(x)$ is the Bessel function of first kind then prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

14 (c) Show that $x^4 = \frac{1}{35}[8P_4(x) + 20P_2(x) + 7P_0(x)]$, where $P_n(x), n = 0,2,4$ are Legendre's polynomials.

5+5+4=14

6. (a) Show that the functions $u(x,y) = e^x \cos y$ is a harmonic and determine its harmonic conjugate $v(x,y)$ and the analytic function $f(z) = u + iv$.

(b) Show that the transformation $w = i\frac{1-z}{1+z}$ transforms the circle $|z| = 1$ onto the real axis of the w-plane and interior of the circle into the upper half of the w-plane.

14

(c) Find the bilinear transformation which maps $z = 1,\ i, -1$ onto the points $w = i, 0, -i$.

5+5+4=14

7. (a) Evaluate using Cauchy's integral formula

$$\oint_C \frac{e^{tz}}{z^2+1}\,dz$$

Where C is the unit circle $|Z| = 3$.

(b) Find the residue of

$$f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$$

at its poles and hence evaluate

$$\oint_C f(z)\,dz$$

Where C is the unit circle $|z| = 2{\cdot}5$.

(c) What type of singularity has the function $f(z) = e^{\frac{1}{z}}/z^2$.

4+5+5=14

$x = e^t$

$\frac{dx}{dt} = e^t$

$\sqrt{\tfrac{1}{2}} = \sqrt{\pi}$

$\Gamma\tfrac{1}{2} =$

$y^2 = v$

$2y\,dy$

$x^2 = u$